

Observational Consequences I – The Extragalactic Dipole

Clock-skew, luminosity-distance anisotropy, and volume growth

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Abstract

Large extragalactic catalogues provide a direct probe of the angular distribution of matter on the past light cone of the observer. Several analyses of radio and infrared surveys have reported dipole amplitudes exceeding the level expected from the kinematic motion of the Solar System with respect to the cosmic rest frame.

Starting from the observable definition of flux-limited source counts, a general first-order kernel is derived that describes how directional perturbations of the past light cone propagate into the dipole amplitude of extragalactic catalogues. The analysis shows that only three physical mechanisms can contribute at leading order: a directional remapping of redshift (“clock-skew”), a directional perturbation of luminosity distance, and a directional perturbation of the shell volume element.

Each mechanism is examined in turn to determine whether it can generate a dipole of order $D \sim 10^{-2}$ without producing pathological behaviour in other observables. The resulting kernel provides a model-independent framework for analysing dipole anisotropies in flux-limited surveys and for testing cosmological models that introduce small directional perturbations of the null-cone geometry.

This framework does not derive those distortions from a specific spacetime metric. Its purpose is to establish the necessary structure through which any geometric cosmology predicting a dipolar distortion of the past light cone must propagate into observable source counts. The implications of these results for the cosmological constant and vacuum-energy interpretation are examined separately in Observational Consequences II.

1. Introduction

Large astronomical catalogues provide a direct probe of the angular distribution of matter on the past light cone of the observer.

Measurements of the dipole anisotropy in extragalactic catalogues have been reported for several large radio and infrared surveys. Analyses of the NRAO VLA Sky Survey (NVSS), the TIFR GMRT Sky Survey (TGSS), and quasar samples constructed from the Wide-field Infrared Survey Explorer (WISE) have consistently found dipole amplitudes exceeding the level expected from purely kinematic motion with respect to the cosmic microwave background frame (e.g. Blake & Wall 2002¹; Singal 2011²; Gibelyou &

¹ Blake, C., and Wall, J., “A velocity dipole in the distribution of radio galaxies,” *Nature* 416, 150–152 (2002). Early analysis of radio-source counts suggesting a dipole consistent with large-scale anisotropy and motivating later catalogue-based studies.

² Singal, A. K., “Large peculiar motion of the solar system from the dipole anisotropy in sky brightness due to distant radio sources,” *The Astrophysical Journal Letters* 742, L23 (2011). Demonstrates that the dipole in radio-source counts appears larger than expected from the CMB kinematic dipole.

Huterer 2012³; Rubart & Schwarz 2013⁴; Tiwari et al. 2015⁵; von Hausegger et al. 2018⁶; Secrest et al. 2022⁷). Although the statistical interpretation of these measurements remains debated, the reported amplitudes typically lie near $D \sim 10^{-2}$, an order of magnitude larger than the simple kinematic expectation.

For a flux-limited survey, the number of sources observed in a direction $\hat{n}$ may be written

$$N(\hat{n}) = \int dz \mathcal{I}(z, \hat{n}),$$

where

$$\mathcal{I}(z, \hat{n}) = \frac{dV}{dz d\Omega}(z, \hat{n}) \Phi(> L_{\min}(z, \hat{n}), z).$$

Here $dV/(dz d\Omega)$ is the differential shell volume at redshift z , $L_{\min}$ is the luminosity threshold implied by the survey flux limit, and $\Phi(> L, z)$ is the cumulative luminosity function of the source population.

Expanding the angular distribution in spherical harmonics, the leading anisotropic contribution is the dipole term

$$N(\hat{n}) = \bar{N}(1 + D\mu),$$

with

$$\mu = \hat{n} \cdot \hat{g},$$

where $\hat{g}$ defines the dipole axis.

In standard cosmology the dominant dipole contribution is expected to arise from the kinematic motion of the observer with respect to the cosmic rest frame.⁸ However, reported dipole amplitudes exceed the level predicted purely by this motion. Although

³ Gibelyou, C., and Huterer, D., “Dipoles in the sky,” *Monthly Notices of the Royal Astronomical Society* 427, 1994–2021 (2012). Examines dipole anisotropies across multiple datasets and discusses the consistency of large-scale anisotropy measurements.

⁴ Rubart, M., and Schwarz, D. J., “Cosmic radio dipole from NVSS and WENSS,” *Astronomy & Astrophysics* 555, A117 (2013). Analyses radio catalogues and finds dipole amplitudes exceeding the simple kinematic expectation.

⁵ Tiwari, P., Kothari, R., Naskar, A., and Nadkarni-Ghosh, S., “Dipole anisotropy in the NVSS radio catalogue,” *Astroparticle Physics* 61, 1–11 (2015). Reanalysis of NVSS source counts confirming an excess dipole relative to the CMB kinematic prediction.

⁶ von Hausegger, S., Rameez, M., and Sarkar, S., “Testing the cosmological principle with radio surveys,” *European Physical Journal C* 78, 914 (2018). Investigates large-scale anisotropy and challenges the assumption of statistical isotropy in cosmological data.

⁷ Secrest, N. J., von Hausegger, S., Rameez, M., et al., “A test of the cosmological principle with quasars,” *The Astrophysical Journal Letters* 937, L31 (2022). Uses WISE-selected quasars to measure an extragalactic dipole whose amplitude exceeds that expected from the Solar System’s motion.

⁸ Ellis, G. F. R., and Baldwin, J. E., “On the expected anisotropy of radio source counts,” *Monthly Notices of the Royal Astronomical Society* 206, 377–381 (1984). Derives the expected dipole in flux-limited radio catalogues produced by the observer’s peculiar motion.

the statistical interpretation of these results remains debated, the reported amplitudes are typically of order

$$D \sim 10^{-2},$$

which motivates examining how such a signal could arise in flux-limited source counts.

Because the count integrand depends only on the redshift mapping, the luminosity threshold, and the shell volume element, any first-order anisotropy of the past light cone must enter through perturbations of these quantities.

The three channels identified here have clear counterparts in standard treatments of line-of-sight perturbations in cosmology, including relativistic formulations of galaxy number counts and weak-lensing theory. Perturbations to the observed redshift, luminosity distance, and volume element are well known to arise from metric and velocity perturbations of the past light cone. The present approach differs in that it does not begin from a perturbed spacetime metric, but from the observable definition of flux-limited source counts, and identifies the minimal set of response functions through which any such perturbations must enter.⁹

Writing the fractional perturbation of the integrand to first order gives

$$\frac{\delta \mathcal{J}}{\mathcal{J}} = -\delta z \frac{d \ln \bar{J}}{dz} - 2\beta(z) \frac{\delta d_L}{d_L} + \frac{\delta V}{V},$$

where

$$\beta(z) = -\frac{\partial \ln \Phi(> L, z)}{\partial \ln L}.$$

Integrating over redshift yields the corresponding dipole amplitude

$$D = \varepsilon \int dz W(z) \left[q(z) \frac{d \ln \bar{J}}{dz} - 2\beta(z) r(z) + s(z) \right].$$

The functions $q(z)$, $r(z)$, and $s(z)$ encode the redshift dependence of perturbations in the redshift mapping, luminosity-distance relation, and shell volume respectively. This expression shows that a dipole in flux-limited catalogues can arise through three physically distinct channels: a directional perturbation of redshift (“clock-skew”), a directional perturbation of luminosity distance, or a directional perturbation of the shell volume element. The geometric interpretation of these three contributions is illustrated schematically in Figure 1 (overleaf), which shows the three mechanisms as distortions of the past light cone along the dipole axis .

⁹ Bonvin, C., and Durrer, R., *What galaxy surveys really measure*, Phys. Rev. D 84 (2011). Relativistic formulation of observed number counts including redshift, lensing, and volume effects.

Challinor, A., and Lewis, A., *The linear power spectrum of observed source number counts*, Phys. Rev. D 84 (2011). Derivation of line-of-sight contributions to observable anisotropies.

The purpose of this paper is to examine these three mechanisms in turn and determine whether they can generate a dipole of order $D \sim 10^{-2}$ without introducing pathological behaviour in other observables. The resulting kernel (a redshift-weighted response function that maps perturbations of the light cone into the observed dipole) provides a general framework for analysing dipole anisotropies in extragalactic catalogues and for evaluating cosmological models that introduce small directional perturbations of the null-cone geometry.

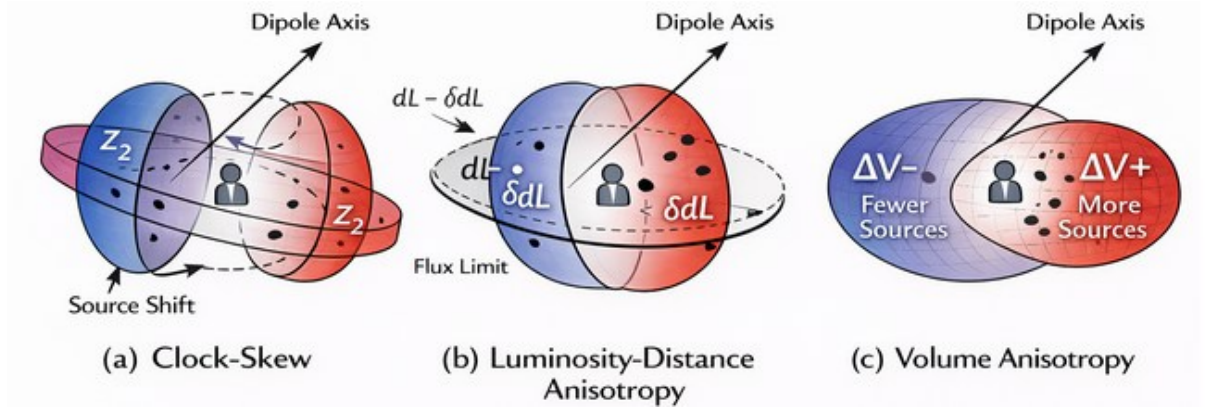


Figure 1.

Schematic illustration of the three first-order mechanisms that generate a dipole in flux-limited extragalactic source counts. Opposing hemispheres are shown in blue and red to indicate lower and higher effective redshift respectively.

(a) Clock-skew: a directional perturbation of the redshift mapping shifts sources between neighbouring redshift shells along the dipole axis.

(b) Luminosity-distance anisotropy: a directional perturbation of the luminosity distance modifies the flux limit surface (shown as the encircling dashed ring), altering the number of sources included above the survey threshold.

(c) Volume anisotropy: a directional perturbation of the differential shell volume changes the number density of sources within the observed shell, producing regions of relative deficit and excess.

Each mechanism corresponds to a distinct term in the dipole kernel derived in the text.

2. Clock-Skew

The first term in the dipole kernel corresponds to a directional perturbation of the observed redshift. At the present level of analysis this term is introduced kinematically, as an observable remapping of the redshift coordinate along the past light cone, rather than derived from an explicit spacetime metric. Its physical admissibility therefore depends on whether such a remapping can be obtained from a covariant geometric perturbation rather than a coordinate artefact. This effect may be represented as a small dipolar remapping of the redshift coordinate,

$$z \rightarrow z + \delta z,$$

with

$$\delta z = \varepsilon q(z) \mu,$$

where $\mu = \hat{n} \cdot \hat{g}$, ε is a dimensionless amplitude, and $q(z)$ describes the redshift dependence of the perturbation.

A directional shift in redshift moves sources between neighbouring shells in the catalogue integral. To first order this produces a fractional perturbation of the count integrand proportional to the redshift gradient of the isotropic distribution. Substituting the above form into the general dipole kernel derived in the Introduction yields

$$D_{\text{cs}} = -\varepsilon \int dz W(z) q(z) \frac{d \ln \bar{j}}{dz}.$$

The clock-skew contribution therefore depends on the logarithmic redshift derivative of the source-count kernel. Physically, the dipole arises because a directional redshift remapping slightly compresses the source distribution in one hemisphere and stretches it in the opposite hemisphere.

The same perturbation necessarily alters the inferred luminosity distance of sources. To leading order the corresponding shift in distance modulus may be written

$$\delta \mu_{\text{mag}} \propto \frac{q(z)}{z} \mu,$$

showing that a pure redshift remapping produces a magnitude anisotropy that grows rapidly at low redshift unless the response function vanishes near the origin.

This behaviour imposes a strong constraint on the clock-skew mechanism.

If $q(z)$ approaches a non-zero constant as $z \rightarrow 0$, the induced magnitude dipole diverges and would already be apparent in local galaxy samples. A viable clock-skew model therefore requires

$$q(z) \rightarrow 0 (z \rightarrow 0).$$

Provided this condition is satisfied, a directional redshift perturbation can generate a dipole in source counts of order

$$D_{\text{cs}} \sim 10^{-2}$$

for modest values of the perturbation amplitude ε . The redshift-remapping channel therefore remains a formally possible contributor to the observed extragalactic dipole at the level of the kernel, although its physical viability depends not only on regular low-redshift behaviour but on derivation from an underlying spacetime geometry.

The next section examines the second channel identified in the dipole kernel, namely a directional perturbation of the luminosity distance.

3. Anisotropic Luminosity Distance

The second term in the dipole kernel arises from a directional perturbation of the luminosity distance. In a flux-limited survey the minimum luminosity detectable at redshift z depends on the inferred luminosity distance through

$$L_{\min}(z) \propto d_L^2(z),$$

so that any anisotropy in d_L modifies the luminosity threshold and hence the number of sources included in the catalogue. To describe this effect, the luminosity distance is written as a small dipolar perturbation of the isotropic background relation,

$$d_L(z, \mu) = \bar{d}_L(z) [1 + \varepsilon r(z) \mu],$$

where $r(z)$ characterises the redshift dependence of the anisotropy and ε is again a dimensionless amplitude.

A change in luminosity distance alters the luminosity threshold of the survey, which in turn changes the cumulative number of sources above that threshold. Using the definition

$$\beta(z) = -\frac{\partial \ln \Phi(> L, z)}{\partial \ln L},$$

the resulting fractional perturbation of the integrand becomes

$$\frac{\delta \mathcal{I}}{\mathcal{I}} = -2\beta(z) \frac{\delta d_L}{d_L}.$$

Substituting the dipolar form of the luminosity-distance perturbation into the general dipole kernel then yields

$$D_L = -2\varepsilon \int dz W(z) \beta(z) r(z).$$

This expression shows that a directional distortion of the luminosity distance modifies source counts through the slope of the luminosity function. Physically, the dipole arises because a small change in the inferred distance shifts the effective luminosity threshold, increasing the number of detectable sources in one hemisphere and decreasing it in the opposite hemisphere.

The same perturbation necessarily produces an anisotropy in the inferred magnitudes of individual sources. To leading order the corresponding shift in distance modulus is

$$\delta \mu_{\text{mag}} = \frac{5}{\ln 10} \varepsilon r(z) \mu \approx 2.17 \varepsilon r(z) \mu.$$

A dipole in source counts of order $D \sim 10^{-2}$ therefore requires

$$\varepsilon r(z) \sim 5 \times 10^{-3}$$

over the effective redshift range of the catalogue.

Unlike the clock-skew mechanism discussed in the previous section, anisotropic luminosity distance does not introduce a singular behaviour at low redshift. However, it predicts a correlated dipole in the magnitudes of sources across the sky. The absence of a strong magnitude dipole in existing samples therefore places constraints on the allowed amplitude of the perturbation. Within these bounds, a modest directional distortion of the luminosity-distance relation provides a natural route for generating a dipole of the observed amplitude.

The final mechanism identified in the dipole kernel involves a directional perturbation of the shell volume element, which is examined in the next section.

4. Anisotropic Volume Growth

The third term in the dipole kernel corresponds to a directional perturbation of the differential shell volume. In the source-count integral, the factor $dV/(dz d\Omega)$ represents the physical volume of the redshift shell observed in a given direction. If the geometry of the past light cone is slightly anisotropic, this volume element may acquire a directional dependence.

This effect is described by writing the differential shell volume as

$$\frac{dV}{dz d\Omega}(z, \mu) = \left(\frac{d\bar{V}}{dz d\Omega} \right)(z) [1 + \varepsilon s(z)\mu],$$

where $s(z)$ characterises the redshift dependence of the perturbation and ε again sets the overall amplitude.

Unlike the mechanisms considered in the previous sections, a perturbation of the shell volume affects the source counts directly, without altering either the redshift mapping or the luminosity threshold.

Substituting the above expression into the general dipole kernel therefore gives

$$D_V = \varepsilon \int dz W(z) s(z).$$

The resulting dipole amplitude is determined by the weighted average of the volume perturbation over the redshift range probed by the catalogue.

Geometrically, a directional perturbation of the shell volume may arise through distortions of either the transverse area of the light cone or its radial depth per unit redshift. If these effects are written in terms of fractional perturbations $u(z)$ and $v(z)$ respectively, the volume response may be expressed schematically as

$$s(z) = 2u(z) + v(z).$$

This decomposition reflects the fact that the shell volume is proportional to the square of the transverse distance scale multiplied by the radial thickness of the shell.

A dipole amplitude of order $D \sim 10^{-2}$ can be produced by this mechanism for modest values of the perturbation amplitude ε , provided that the volume response $s(z)$ remains of order unity over the effective support of the catalogue. However, the geometric origin of such a perturbation must ultimately be consistent with the behaviour of other observables on the past light cone. In particular, distortions of the transverse or radial distance scales generally induce correlated perturbations in luminosity distance or redshift mapping. For this reason the volume-growth channel is not completely independent of the mechanisms discussed in Sections 2 and 3, and its physical interpretation must be considered within a consistent geometric framework.

Having examined the three contributions that appear in the dipole kernel, their properties can now be compared. The alternative mechanisms are assessed to show how they can plausibly account for the observed amplitude of the extragalactic dipole.

5. Comparative Evaluation of the Dipole Mechanisms

The analysis of the preceding sections shows that three distinct mechanisms can generate a dipole in flux-limited source counts at first order: directional redshift remapping (clock-skew), anisotropic luminosity distance, and anisotropic shell volume. Each corresponds to a different term in the dipole kernel derived earlier.

Although these mechanisms appear formally similar in the kernel expression,

$$D = \varepsilon \int dz W(z) \left[q(z) \frac{d \ln \bar{J}}{dz} - 2\beta(z)r(z) + s(z) \right],$$

their physical consequences differ in several important respects.

The clock-skew contribution depends on the redshift derivative of the source-count kernel. The dipole therefore arises from a directional compression or stretching of the redshift distribution of sources. This mechanism is particularly sensitive to the behaviour of the perturbation function $q(z)$ at low redshift. Unless $q(z)$ vanishes as $z \rightarrow 0$, the induced perturbation produces a divergent magnitude anisotropy in nearby samples. Clock-skew models must therefore satisfy a strong regularity condition near the origin.

The luminosity-distance mechanism operates through a different route. A directional perturbation of d_L alters the luminosity threshold implied by the survey flux limit, thereby changing the number of sources included in the catalogue. In this case the dipole amplitude depends on the slope of the luminosity function through the parameter $\beta(z)$. Because the required amplitude $D \sim 10^{-2}$ can be obtained for modest values of the perturbation $\varepsilon r(z)$, this mechanism provides a comparatively efficient

route for generating the observed dipole. Its principal observational consequence is the prediction of a correlated magnitude dipole across the sky.

The volume-growth mechanism modifies the source counts directly through the shell volume element. In contrast to the previous two mechanisms, it does not rely on either the redshift gradient of the catalogue or the luminosity-function slope. Instead, the dipole amplitude is determined by the weighted average of the volume response function $s(z)$. This makes the mechanism mathematically straightforward to realise, but also less specific, because distortions of the shell volume generally arise from the same geometric perturbations that affect luminosity distance or radial redshift mapping.

These differences can be summarised schematically by examining the structure of the three contributions to the dipole kernel:

- clock-skew couples to the redshift gradient of the catalogue distribution;
- luminosity-distance anisotropy couples to the luminosity-function slope;
- volume anisotropy couples directly to the geometry of the light-cone volume.

From a phenomenological perspective, the luminosity-distance channel provides the most natural mechanism for producing a dipole of the observed amplitude without introducing pathological behaviour at low redshift. The clock-skew mechanism remains viable but requires a carefully regulated redshift response, while the volume-growth channel is geometrically broader and may overlap with the other two effects once a specific spacetime perturbation is specified.

The central conclusion of this comparison is therefore not that a single mechanism must operate in isolation, but that any geometric explanation of the extragalactic dipole must manifest itself through one or more of these three channels. The kernel derived in this paper thus provides a general framework for analysing dipole anisotropies independent of any specific cosmological model.

The next section considers the implications of this framework for geometrical models in which the past light cone itself possesses a small dipolar anisotropy.

6. Geometric Interpretation

The preceding analysis has been deliberately independent of any specific cosmological model. Starting from the observable definition of flux-limited source counts, a dipole anisotropy in extragalactic catalogues arises through three first-order channels: directional perturbations of redshift, luminosity distance, or shell volume. Each channel corresponds to a distinct contribution to the dipole kernel derived in Section 1.

The structure of the kernel implies a general conclusion. Any spacetime geometry that introduces a small dipolar perturbation of the past light cone will produce observable consequences in source counts through one or more of these channels. In other words,

dipole amplitudes exceeding the kinematic expectation follow generically from directional distortions of the null-cone geometry connecting the observer to distant sources.

In practical terms, such distortions may appear as perturbations of the redshift–distance relation, directional variations in inferred luminosity distance, or anisotropic growth of the shell volume element. The analysis of Sections 2–4 shows how each of these effects propagates into the observable dipole amplitude.

This perspective provides a natural context in which to examine geometric cosmological models whose observable consequences differ slightly from those of perfectly isotropic backgrounds. One example is the Time-Symmetric Parity Constraint (TSPC), which introduces an algebraic restriction on the spacetime metric capable of producing small directional perturbations of the null-cone structure. Within such a framework, the functions $q(z)$, $r(z)$, or $s(z)$ appearing in the dipole kernel may be derived from the underlying geometry rather than introduced phenomenologically.

The present analysis does not attempt to derive these functions explicitly for any particular model. Instead, it establishes the observational pathways through which geometric perturbations must manifest themselves. Any theory predicting a dipolar distortion of the past light cone must therefore satisfy the constraints identified in the previous sections: the perturbation must generate a dipole amplitude of order $D \sim 10^{-2}$ without producing pathological behaviour at low redshift or violating existing constraints on magnitude anisotropies.

In this sense the dipole kernel derived here may be viewed as a diagnostic tool. It provides a model-independent framework for evaluating whether a given geometric perturbation is capable of producing the observed extragalactic dipole.

The final section summarises these results and outlines the observational conditions under which geometric explanations of the dipole may be tested.

7. Conclusions

Large extragalactic catalogues provide a direct probe of the angular distribution of matter on the past light cone of the observer. The leading anisotropic component of this distribution is the dipole term in the source-count expansion. Several analyses of radio and quasar catalogues have reported dipole amplitudes exceeding the level expected from purely kinematic motion with respect to the cosmic rest frame, although the statistical interpretation of these results remains the subject of continuing investigation.¹⁰

¹⁰ Nicholas J. Secrest, et al. *A Test of the Cosmological Principle with Quasars*. *The Astrophysical Journal Letters* **937**, L31 (2022). Measures the dipole anisotropy of a large WISE-selected quasar sample, reporting an amplitude exceeding that expected from the Solar System's kinematic motion with respect to the cosmic microwave background. /cont ...

Starting from the observable definition of flux-limited source counts, this paper has derived a general first-order kernel describing how directional perturbations of the past light cone propagate into the dipole amplitude. The analysis shows that only three physical mechanisms can contribute at leading order: a directional remapping of redshift (clock-skew), a directional perturbation of luminosity distance, and a directional perturbation of the shell volume element.

Each mechanism produces a characteristic contribution to the dipole kernel,

$$D = \varepsilon \int dz W(z) \left[q(z) \frac{d \ln \bar{j}}{dz} - 2\beta(z)r(z) + s(z) \right],$$

which provides a model-independent framework for analysing dipole anisotropies in flux-limited catalogues.

The comparative evaluation presented here shows that a dipole of order $D \sim 10^{-2}$ can arise through modest perturbations of the luminosity-distance relation or the shell-volume element, while clock-skew models must satisfy stronger constraints on their low-redshift behaviour.

The significance of this result lies in its geometric interpretation. Any cosmological model that introduces a small dipolar perturbation of the null-cone structure connecting the observer to distant sources will generically generate observable effects in source counts through one or more of these channels.

TSPC, as developed in earlier papers of this series, introduces an algebraic restriction on the spacetime metric capable of producing directional perturbations of the past light cone. Although the present work does not derive the dipole response functions directly from that geometry, the kernel derived here provides a framework within which such predictions may be tested.

In this sense the analysis represents a step from conceptual formulation toward observational evaluation. The earlier papers in the TSPC series developed the theoretical basis of the parity constraint and examined the limitations of the standard cosmological model. The present work identifies one concrete observational pathway through which geometric perturbations associated with the constraint could manifest themselves in astronomical data.

Future work will therefore focus on deriving the response functions $q(z)$, $r(z)$, and $s(z)$ directly from the underlying spacetime geometry, thereby replacing the phenomenological kernel developed here with one obtained from first principles. The

/ continued... Manuel Rubart & Dominik J. Schwarz. *Cosmic Radio Dipole from NVSS and WENSS*. *Astronomy & Astrophysics* 555, A117 (2013). Analysis of large radio-source catalogues reporting dipole amplitudes larger than the simple kinematic expectation and motivating continued investigation of large-scale dipole anisotropies

resulting framework may then be tested against existing and forthcoming large-scale surveys.

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