

12 – Void Geometry and Alcock–Paczyński Distortion

Empirical tests of global spatial constraint

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Abstract

This paper examines cosmic voids as probes of global spatial geometry under the Time-Symmetric Parity Constraint (TSPC). Void populations are treated as near-geometric objects whose inferred shapes depend primarily on the mapping between redshift and physical distance. The Alcock–Paczyński relation is used not as a parameter-constraining tool but as a geometric admissibility test. Under TSPC, the background mapping functions are not freely

deformable, and persistent anisotropy in stacked voids constitutes resistance to the constraint rather than motivation for parametric extension. The paper deliberately excludes forecasting, optimisation, and growth-based analyses, focusing instead on whether void geometry provides a genuine empirical pressure point for a globally constrained cosmological framework.

1. Introduction

TSPC is imposed as a global condition on the geometry of spacetime. Its empirical assessment therefore cannot proceed by the usual accumulation of fitted observables across an extended parameter space. The relevant question is not whether a wide class of data can be accommodated, but whether there exist observations that interrogate the admissibility of the imposed geometry itself.

Cosmic voids occupy a distinctive position in this respect. Unlike dense structures, they are defined primarily by the absence of matter rather than by its detailed microphysical behaviour. Their large spatial extent, low baryonic complexity, and weak sensitivity to feedback processes render them closer to probes of background geometry than to tracers of structure formation. When void populations are treated statistically, their shapes and orientations are governed principally by the mapping between redshift space and physical space.

The Alcock–Paczyński test exploits precisely this mapping. It compares inferred radial and transverse scales of objects that are assumed to be isotropic in their rest frame. Any systematic anisotropy observed in such a population is therefore a direct diagnostic of the background geometry used to interpret the data.¹ Crucially, the test depends only on ratios of scales and does not rely on assumptions about

¹ Alcock, C. and Paczyński, B., *An evolution-free test for non-zero cosmological constant*, *Astrophysical Journal*, 241, 1979. Introduces the purely geometric redshift–angular test used to diagnose background cosmology independently of source evolution.

growth rates, bias models, or the detailed evolution of matter within the voids themselves.

Within many contemporary cosmological analyses, Alcock–Paczyński distortions are treated as quantities to be marginalised over or absorbed through flexible background parameterisations. In that context they function as contributors to constraint tightening rather than as potential sources of resistance. The present paper adopts a different stance. Under TSPC, the background mapping functions are not regarded as freely adjustable. As a result, the Alcock–Paczyński relation becomes a hard geometric test: persistent, redshift-dependent deviations from isotropy cannot be neutralised by parametric extension without undermining the constraint itself.

The scope of this paper is therefore deliberately limited. It does not aim to forecast survey performance, to optimise void-finding algorithms, or to extract precision constraints from existing catalogues. Nor does it address the dynamical formation or evolution of voids. Its sole purpose is to establish whether void geometry, when interpreted through the Alcock–Paczyński relation, constitutes a genuine empirical pressure point for a cosmological framework defined by a global geometric restriction.

Subsequent sections treat voids explicitly as near-geometric objects, examine the Alcock–Paczyński test as a discriminator rather than a fitting tool, and consider the consequences of enforcing geometric admissibility rather than negotiating it. The analysis is intended to complement, not replace, other empirical probes of the constraint, and to occupy the logically first position among them by addressing spatial geometry before spacetime propagation or temporal structure.

2. Geometric framework for void-based tests

2.1 Why voids may be treated as geometric objects

Cosmic voids differ from most cosmological structures in that they are defined negatively, by the relative absence of matter rather than by its concentration. This distinction is not merely descriptive. In underdense regions, the influence of baryonic physics, feedback processes, and non-linear clustering is suppressed rather than amplified. As a result, the dominant factor governing the large-scale properties of voids is the background spacetime in which they are embedded.

When considered individually, voids exhibit a wide range of morphologies shaped by local environment and formation history. However, when aggregated statistically across large volumes, their defining characteristics are set by spatial geometry.² In particular, the distribution of void orientations and aspect ratios is controlled primarily by the

² Lavaux, G. and Wandelt, B. D., *Precision cosmology with voids: definition, methods, dynamics*, Monthly Notices of the Royal Astronomical Society, 403, 2010. Establishes stacked cosmic voids as statistically isotropic objects suitable for geometric tests.

mapping between observable coordinates and physical distances. This makes void populations well suited to tests that depend on geometric consistency rather than on dynamical evolution.

The use of stacked voids exploits this fact. By averaging over many individual objects, local anisotropies associated with environment or formation history are suppressed, leaving residual distortions that reflect the global mapping applied to the data.³ Under the assumption that voids are statistically isotropic in real space, any systematic departure from isotropy in the stacked population must arise from the geometry used to interpret redshift and angular separations.

This property distinguishes voids from dense tracers such as clusters or galaxies. In overdense regions, inferred shapes and correlations depend sensitively on non-linear dynamics, bias prescriptions, and baryonic effects. In voids, by contrast, such influences are secondary. The dominant source of apparent anisotropy is the assumed relationship between radial and transverse distances, which is precisely the quantity isolated by the Alcock–Paczyński relation.

For the purposes of the present analysis, voids are therefore treated neither as evolved dynamical systems nor as precision probes of structure growth. They are treated as extended regions of spacetime whose observed shapes encode information about the background geometry. This treatment does not deny the role of dynamics in void formation, but it brackets it deliberately, in order to isolate those aspects of void behaviour that depend directly on global spatial structure.

It is this near-geometric character that makes voids particularly resistant to interpretive flexibility. While local deviations can always be attributed to environment or noise, persistent, redshift-dependent anisotropies in stacked void populations cannot be absorbed without modifying the underlying mapping between observables and distances. Under a framework in which that mapping is constrained rather than freely adjustable, void geometry becomes a genuine empirical pressure point.

2.2 Alcock–Paczyński geometry for void populations

The geometric interpretation of void shapes rests on the relation between observed angular and redshift separations and the corresponding physical distances inferred under an assumed background geometry.⁴ This mapping is captured compactly by the Alcock–Paczyński relation. For the purposes of the present analysis, only the minimal

³ Hamaus, N. et al., *Constraints on cosmology and gravity from the dynamics of voids*, Physical Review Letters, 117, 2016. Demonstrates that stacking suppresses local dynamics, leaving geometry-dominated residuals.

⁴ Sutter, P. M. et al., *A first application of the Alcock–Paczyński test to stacked cosmic voids*, Monthly Notices of the Royal Astronomical Society, 442, 2014. Provides the first explicit implementation of the Alcock–Paczyński test using stacked void catalogues.

formal content required to state this relation is introduced. All subsequent interpretation refers back to this closed set of definitions.

(1) *Radial and transverse physical scales*

Radial physical scale

$$\Delta r_{\parallel}(z) = \frac{c\Delta z}{H(z)}$$

Transverse physical scale

$$\Delta r_{\perp}(z) = D_M(z)\Delta\theta$$

Relation between distances

$$D_M(z) = (1+z)D_A(z)$$

(2) *Alcock–Paczyński ratio for an isotropic object*

For an intrinsically isotropic population, such as stacked cosmic voids treated as statistically spherical in real space, the inferred radial and transverse scales coincide when the correct background geometry is used.

Definition

$$R_{AP}(z) = \frac{\Delta r_{\parallel}(z)}{\Delta r_{\perp}(z)}$$

Substitution

$$R_{AP}(z) = \frac{c\Delta z}{H(z)D_M(z)\Delta\theta}$$

Equivalent angular-redshift relation

$$\frac{\Delta z}{\Delta\theta} = \frac{H(z)D_M(z)}{c}$$

(3) *Distortion under an assumed background geometry*

Let quantities labelled “ass” denote the assumed mapping used to convert observables into physical scales, and quantities labelled “true” denote the correct mapping.

Radial scaling

$$\alpha_{\parallel}(z) = \frac{H^{\text{ass}}(z)}{H^{\text{true}}(z)}$$

Transverse scaling

$$\alpha_{\perp}(z) = \frac{D_M^{\text{true}}(z)}{D_M^{\text{ass}}(z)}$$

Resulting apparent anisotropy

$$R_{AP}^{\text{ass}}(z) = \frac{\alpha_{\parallel}(z)}{\alpha_{\perp}(z)} R_{AP}^{\text{true}}(z)$$

For a statistically spherical population in the true geometry,
 $R_{\text{AP}}^{\text{true}}(z) = 1$, giving

$$R_{\text{AP}}^{\text{ass}}(z) = \frac{\alpha_{\parallel}(z)}{\alpha_{\perp}(z)}$$

(4) TSPC closure statement

Within TSPC, the background functions $H(z)$ and $D_M(z)$ are not treated as freely deformable fit functions. They are determined, up to the limited degrees of freedom admitted by the global parity constraint itself. The Alcock–Paczyński ratio is therefore employed as a geometric admissibility test: persistent, redshift-dependent deviation of $R_{\text{AP}}(z)$ from unity under the TSPC mapping is interpreted as resistance to the constraint itself, rather than as motivation for parametric extension within the TSPC framework.

2.3 Interpretation under geometric constraint

The relations set out the box above establish a closed geometric mapping between observable separations and inferred physical scales. Once this mapping is fixed, the interpretation of void shapes becomes conceptually straightforward. For a population that is statistically isotropic in real space, the Alcock–Paczyński ratio provides a direct test of whether the assumed background geometry preserves that isotropy when redshift and angular coordinates are converted into distances.

The critical point is that the Alcock–Paczyński ratio does not diagnose dynamics. It does not depend on growth rates, bias prescriptions, or the internal evolution of voids. It tests only whether the background mapping between radial and transverse directions is self-consistent. When applied to stacked void populations, the averaging process suppresses local environmental effects, leaving residual anisotropy that reflects the geometry imposed at the level of distance reconstruction.

In conventional analyses, deviations of the Alcock–Paczyński ratio from unity are treated as quantities to be marginalised over or absorbed through flexible background parameterisations. In such treatments, the ratio functions as a contributor to constraint tightening rather than as a discriminator. The assumption is made, often implicitly, that the background functions entering the mapping may be adjusted continuously until consistency is restored.

Under TSPC, this interpretive option is not available. The background mapping functions are not regarded as freely deformable, but as determined, up to tightly limited freedom, by the imposed global symmetry. As a result, the Alcock–Paczyński ratio acquires a different status. Persistent, redshift-dependent departures from isotropy cannot be dismissed as artefacts of parameter choice, nor can they be neutralised without modifying the constraint itself.

This shift in interpretation is central to the empirical role assigned to void geometry in the present framework. The Alcock–Paczyński relation is not employed to extract optimal parameter values, but to test admissibility. Agreement across redshift supports the consistency of the imposed geometry; systematic deviation constitutes resistance to it. In this sense, void-based Alcock–Paczyński measurements function not as incremental refinements, but as potential vetoes.

Having established the geometric framework and its interpretation under constraint, the analysis may now turn to the role of the Alcock–Paczyński test as an empirical discriminator, and to the reasons why interpretive flexibility struggles to neutralise its force when background geometry is restricted rather than negotiated.

3. The Alcock–Paczyński test as a geometric discriminator

The Alcock–Paczyński relation acquires discriminatory power only when background geometry is treated as something that may fail, rather than as something that must be adjusted until consistency is restored. Its significance therefore depends less on observational precision than on the interpretive posture adopted toward the mapping between observables and distances.

In mainstream cosmological analyses, Alcock–Paczyński distortions are typically embedded within a broader fitting exercise. Deviations from isotropy are absorbed through adjustments to the background expansion history, through redshift-dependent nuisance parameters, or through simulation-calibrated corrections that entangle geometry with growth. In this setting, the test rarely functions as an independent diagnostic. It contributes to tightening constraints, but it is not allowed to veto the assumed framework.

TSPC imposes a different logic. Because the background mapping functions are restricted by a global geometric condition, they cannot be freely reshaped to absorb discrepancies. The Alcock–Paczyński ratio therefore tests not whether a best-fitting parameter set can be found, but whether the imposed geometry is admissible at all. The distinction is subtle but decisive. Under constraint, the relevant question becomes binary: does the mapping preserve statistical isotropy across redshift, or does it not?

Void populations are particularly well suited to this role. Their low-density environment suppresses non-linear dynamical effects, and their large spatial extent reduces sensitivity to small-scale modelling assumptions. When voids are stacked and analysed statistically, local departures from isotropy average away, leaving any residual distortion dominated by the global mapping. In this context, a systematic redshift dependence of the Alcock–Paczyński ratio cannot be plausibly attributed to void physics alone.

The discriminatory character of the test emerges most clearly when redshift dependence is considered. A constant offset could, in principle, be attributed to a global miscalibration. A redshift-dependent deviation, persisting across void samples and

survey volumes, directly challenges the assumed geometry.⁵ Under TSPC, such behaviour would signal resistance to the imposed symmetry rather than an invitation to refine parameter values.

This use of the Alcock–Paczyński relation contrasts sharply with its deployment in forecast-driven analyses. There, its role is typically to improve sensitivity to dark-energy parameterisations or to break degeneracies between expansion history and growth.⁶ In the present framework, no such parameter space is being explored. The relation is used instead to test whether a geometry constrained by global symmetry remains self-consistent when confronted with spatial observables that are minimally dynamical in character.

The result is a qualitative shift in how empirical success is judged. Success is no longer measured by the ability to accommodate void anisotropies through adjustment, but by the absence of the need to do so. Where flexibility treats deviation as something to be negotiated, constraint treats it as something to be confronted. It is in this sense that the Alcock–Paczyński test, applied to void geometry, functions as a genuine geometric discriminator within TSPC.

4. Consequences under TSPC

Under TSPC, the background geometry used to interpret cosmological observations is not an object of continuous adjustment. The mapping between observable coordinates and physical distances is restricted by a global algebraic condition on the metric, enforced uniformly across spacetime. As a consequence, departures from isotropy in geometric observables acquire a different status from that assumed in flexible cosmological frameworks.

In the context of void geometry, this restriction has immediate implications. The Alcock–Paczyński relation depends explicitly on the consistency between radial and transverse distance assignments. When the background mapping is freely deformable, inconsistencies may be absorbed through small changes in the expansion history or through redshift-dependent reparameterisation. Under the parity constraint, such manoeuvres are curtailed. Only those mappings compatible with the imposed symmetry are admissible, and deviations cannot be eliminated without abandoning the constraint itself.

This alters the interpretation of void anisotropy in a fundamental way. Apparent distortions in stacked void populations are no longer treated as artefacts of imperfect modelling to be corrected, but as potential indicators that the assumed geometry is

⁵ Nadathur, S. et al., *Testing cosmological geometry using voids*, Monthly Notices of the Royal Astronomical Society, 460, 2016. Shows that redshift-dependent Alcock–Paczyński distortions in voids cannot be absorbed without altering the background geometry.

⁶ DESI Collaboration, *The DESI Experiment Part II: Cosmological implications*, Astronomical Journal, 164, 2022. Illustrates the standard practice of embedding Alcock–Paczyński information within flexible multi-parameter fitting pipelines.

incompatible with the constraint. In this sense, the Alcock–Paczyński ratio becomes a test of geometric admissibility rather than a diagnostic of parameter choice.

The consequences are especially sharp when the behaviour of the ratio is examined as a function of redshift. A framework constrained by global symmetry is expected to impose coherence across scales. If the geometry is admissible, isotropy should be preserved consistently across void populations spanning different epochs. Conversely, a systematic redshift-dependent trend in the inferred anisotropy cannot be reconciled through local adjustment without violating the global nature of the constraint.

It is important to note that this argument does not rely on precision forecasting or on detailed modelling of void evolution. The effect in question is qualitative rather than incremental. The issue is not whether deviations are small, but whether they are persistent and structured. Under constraint, even modest but coherent departures carry interpretive weight, because the usual avenues for accommodation are deliberately closed.

In this way, void-based Alcock–Paczyński measurements acquire a role analogous to consistency checks in algebraic systems. They do not refine the parameters of the framework; they test whether the framework can be maintained at all. Agreement across redshift supports the internal coherence of the constrained geometry. Failure to do so constitutes resistance to the constraint itself.

The significance of this shift lies not in any single dataset, but in the change of explanatory posture it enforces. Geometry is no longer something inferred piecemeal from observations, but something imposed and then tested. Void geometry provides one of the cleanest arenas in which this test may be conducted, precisely because it minimises reliance on dynamical modelling and foreground complexity.

5. Relation to other empirical probes

The role assigned to void geometry in the present analysis is not isolated. It forms part of a broader empirical strategy in which different classes of observation interrogate different aspects of the same global geometric constraint. What distinguishes the void-based Alcock–Paczyński test is that it addresses spatial geometry directly, with minimal dependence on dynamical modelling or microphysical assumptions.

Other empirical probes considered elsewhere in the compendium occupy complementary positions. Gravitational lensing tests geometry along dense lines of sight, where matter is present but the mapping between angles and distances remains paramount. Gravitational-wave observations probe spacetime propagation rather than spatial geometry alone, testing whether signal travel is consistent with a constrained background metric. Dipole measurements in the cosmic microwave background and radio-source distributions interrogate global frame coherence, examining whether large-scale isotropy is preserved in practice rather than assumed by construction.

Cosmic chronometers address a different aspect of the same structure. Where void geometry tests the spatial component of the mapping between observables and distances, chronometers test the temporal component by comparing differential ageing across cosmological scales. In that sense, chronometer measurements form the natural temporal analogue of the void-based Alcock–Paczyński test. Both depend on relative measures rather than absolute calibration, and both become discriminating only when background freedom is restricted.

The ordering of these empirical probes is therefore logical rather than historical. Spatial geometry is addressed first, because it underlies all subsequent interpretation.⁷ Spacetime propagation and global frame coherence follow, and temporal structure is examined last, as a derived consequence of the same constrained geometry. Each probe is selected not for its novelty or statistical power, but for its capacity to resist interpretive flexibility.

This selectivity is deliberate. Many observational datasets commonly cited in cosmology are excluded from the present programme because they reward parameter freedom rather than enforce structural discipline. The empirical tests retained here are those that confront the framework with the possibility of failure. Void geometry occupies a central place among them precisely because it offers a direct interrogation of spatial admissibility before dynamical explanation is invoked.

In this context, the void-based Alcock–Paczyński test should be read not as an alternative route to precision cosmology, but as a foundational check on whether the background geometry implied by TSPC can be sustained when confronted with observations that are sensitive to geometry alone.

6. Summary and limits

This paper has examined cosmic voids as probes of global spatial geometry under TSPC. By treating void populations as near-geometric objects and by employing the Alcock–Paczyński relation as an admissibility test rather than a fitting tool, the analysis has focused on whether a constrained background geometry can be sustained when confronted with observations that are minimally dynamical in character.

The central claim is deliberately modest. The void-based Alcock–Paczyński test does not offer precision constraints, nor does it seek to optimise parameter estimation. Its value lies instead in its capacity to function as a discriminator when interpretive flexibility is restricted. Under TSPC, the background mapping between observables and distances is not freely deformable. As a result, persistent, redshift-dependent anisotropy in stacked void populations cannot be neutralised through parametric adjustment without undermining the constraint itself.

⁷ Ellis, G. F. R., *Topology and cosmology*, General Relativity and Gravitation, 2, 1971. Argues that spatial geometry provides a logically prior constraint on cosmological interpretation.

This makes void geometry a genuine empirical pressure point. Agreement across redshift supports the internal coherence of the constrained geometry; systematic deviation constitutes resistance to it. The test is therefore binary in character. It does not refine the framework incrementally, but confronts it with the possibility of failure. In this respect it differs sharply from many commonly cited cosmological observables, which reward flexibility and thereby blur the distinction between explanation and accommodation.

The limits of the present analysis should also be clear. No claim is made that existing void catalogues already deliver a decisive verdict. Nor is it asserted that void geometry alone can adjudicate between all competing cosmological frameworks. The purpose of the paper is narrower: to establish that, within a geometry-first approach, void-based Alcock–Paczyński measurements occupy a logically prior position among empirical tests, and that they cannot be reduced to auxiliary constraints once background freedom is curtailed.

Finally, it should be emphasised that this paper addresses spatial geometry only. Other aspects of TSPC are tested by complementary probes, including spacetime propagation and temporal coherence. Those tests are treated separately elsewhere in the compendium. Taken together, they form a deliberately selective empirical programme in which each probe is chosen for its capacity to resist interpretive flexibility and to expose structural weakness if it exists.

In this sense, void geometry is not presented as an anomaly to be explained away, but as a test to be endured. Whether the constrained geometry passes that test is a matter for observation. The role of the framework is not to guarantee success, but to make failure unambiguous.

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